WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 6th Semester Examination, 2023

MTMGDSE03T-MATHEMATICS (DSE2)

Time Allotted: 2 Hours   Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: 2×5 = 10

    (a) Define the operators Δ and ∇, and find a relation between them.

    (b) Show that the average operator μ is given by

    $$\mu \equiv \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right), \text{ where } E \text{ is the shift operator.}$$

    (c) Prove that $\left(\dfrac{\Delta^2}{E} \right) x^3 = 6x$, where the interval of differencing is taken as 1.

    (d) Why does the Newton-Raphson method for computing a simple root of an equation $f(x) = 0$ call method of tangents?

    (e) Use Simpson's one-third rule to evaluate the integral $\int\limits_{0}^{2} (x^2 + 3)dx$, taking two equal subintervals.

    (f) What is the condition of convergency of Gauss-Seidel iteration method to solve the system of n linear simultaneous equations? Is this condition both necessary and sufficient?

    (g) Use Euler's method to find $y(0.2)$ by solving the following initial value problem:
    $$\frac{dy}{dx} = y, \ y(0) = 1 \ (\text{use } h = 0.1)$$

    (h) Is the following system of equations diagonally dominant? If not make it diagonally dominant.
    $$3x + 4y + 15z = 54$$
    $$x + 12y + 3z = 39$$
    $$10x + y - 2z = 7$$

2. (a) A certain function $f(x)$ is defined on the interval [0, 1] in such a way that   4

    $$f(0) = 0, \ f\left(\frac{1}{2}\right) = -1, \ f(1) = 0.$$

    Find the quadratic polynomial $p(x)$ which agrees with $f(x)$ for $x = 0, \frac{1}{2}, 1$

    (b) Find the missing term in the following table: 4

x:	0	1	2	3	4
$f(x)$:	1	3	9	?	81

3. (a) Use Lagrange interpolation formula to construct the interpolating polynomial from the following tabulated data:

$$x: \quad 0 \quad 2 \quad 3 \quad 4$$
$$y: \quad 4 \quad 26 \quad 58 \quad 112$$

6

(b) Show that the third differences of a quadratic function are zero.

2

4. (a) Evaluate $\int_2^3 \dfrac{dx}{1+2x}$ by Trapezoidal Rule taking six equal sub intervals and find the result correct upto four decimal places.

4

(b) If $f(x)$ is a polynomial of degree two, prove that

$$\int_0^1 f(x)dx = \frac{1}{12}[5f(0) + 8f(1) - f(2)]$$

4

5. Solve the following system of equations by LU-Factorization method:

$$2x - 3y + 10z = 3,$$
$$-x + 4y + 2z = 20,$$
$$5x + 2y + z = -12$$

8

6. Describe Successive Over Relaxation (SOR) method for numerical solution of a system of linear equations. Write down the condition of convergency for any initial condition.

6+2

7. (a) Using Euler's method, find the solution of the differential equation

$$\frac{dy}{dx} = x^2 - y, \quad y(0) = 1$$

for $x = 0.4$, taking $h = 0.1$. Compare the result with the exact solution.

4+2

(b) Give the geometrical interpretation of Trapezoidal rule for numerical integration.

2

8. (a) From the equation $x^5 - a = 0$, deduce iterative formula by using Newton-Raphson method as

$$x_{n+1} = \frac{1}{5}\left[4x_n + \frac{a}{x_n^4}\right], \quad n = 0, 1, 2, \cdots.$$

Use the method to find $\sqrt[5]{3}$ correct upto two significant figures.

4+2

(b) Find the iterative function of the following equation $e^x - 4x = 0$ for which the fixed point iteration method is convergent in the interval $[2, 3]$.

2

9. (a) Compute a real root of the following equation between 2 and 3 by using bisection method: $x^3 - 4x - 9 = 0$, correct upto two decimal places.

5

(b) Find $\Delta^4 f(x)$, where $f(x) = (3x+2)(x-2)(x+1)(5x-1)$ and the interval of differencing is unity.

3

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